



**Prize Winner**

**Programming, Apps &  
Robotics  
Year 9-10**

**Vinuka Kaluwila**

**Pembroke School - Middle School**



# Simulating Pandemics – Vinuka Kaluwila

## Introduction

In early 2020, the world was shaken, as an unknown virus - COVID-19 - erupted across the globe. While arguably this pandemic is still ongoing, it's safe to say that the majority of infected cases are in the past. But this raises the question – how exactly, do pandemics evolve over time?

## Aim and Scientific Purpose

This project aims to educate people about some of the interesting maths and science behind the evolution of a pandemic; through an interactive simulation where users can see how a pandemic change over time, by changing the initial conditions.

## Potential Uses

This simulation can be used in education institutions, as a teaching resource to show students the intricacies of how we predict pandemics, as well as to teach the broader topic of differential equations to students.

## Requirements to Run the Program

Desmos online graphing calculator, the platform on which this program was built, is compatible with most online browsers, including the latest two versions of Chrome, Firefox, Safari, and Microsoft Edge. This is a free website which the public can easily access, hence why this platform was chosen.

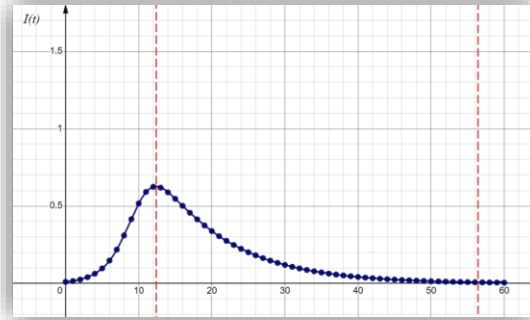
## Instructions for Use

1. Open <https://www.desmos.com/calculator/siyxib0f5v>.
2. Set the initial conditions.  $I_0$  refers to the fraction of the population initially infected,  $\beta$  refers to the probability of contracting the disease, and  $\gamma$  refers to the probability of recovering from the disease. By default, the graph will be set to  $I_0 = 0.01$ ,  $\beta = 0.7$  and  $\gamma = 0.1$ . This can be done using the sliders, as shown in Figure 01.



7. The user can view the maximum of the function, (basically when the most people are infected) in the “Local Maxima” folder. This shows where the derivative of the function is 0. This usually also brings up other erroneous points due to the nature of the approximation, hence it is up to the human user to figure out which line corresponds to the highest point. For example, as can be seen in Figure 05, only the red line at the center of the image corresponds to the highest value of  $I(t)$ , whereas the line to the very right of the image is falsely showing

Figure 05: Showing erroneous solutions do the highest points on the graph.



6. If there are any questions, there are instructions on the program itself for the user. Similarly, feel free to contact me on [vinuka.kaluwila@pembroke.sa.edu.au](mailto:vinuka.kaluwila@pembroke.sa.edu.au).

### Full Code and Explanation

What makes simulating pandemics interesting, is that they are governed by what’s known as “differential equations” rather than regular “equations.” The difference between these two terms, is that a differential equation tells us how a function of time *changes* rather than what it is – where  $\frac{d}{dt}$  is the “derivative” or the rate of change of the function at any given point  $t$ . It is the same forms of equations which dictate many other things in the universe, such as the movement of planets, weather patterns, and fluid flow.

The differential equations required to model the simplest form of a pandemic are:

$$\begin{aligned}\frac{d}{dt}S(t) &= -\beta S(t)I(t) \\ \frac{d}{dt}I(t) &= \beta S(t) - \gamma I(t) \\ \frac{d}{dt}R(t) &= \gamma I(t)\end{aligned}$$

In this system,  $t$  refers to time, which is the variable graphed on the  $x$ -axis on the final project.  $S(t)$ ,  $I(t)$ , and  $R(t)$  are functions of time; of which  $S(t)$  is the fraction of the population susceptible of getting the disease,  $I(t)$  is the fraction of the population infected with the disease, and  $R(t)$  are the fraction of the population recovered from the disease. Note that  $S(t) + I(t) + R(t) = 1$ . In the final project,  $I(t)$  was what was graphed on the  $y$ -axis, as it is arguably the most important variable.

The problem with differential equations, however, is that there are sometimes **no analytic solutions** to these equations. This means that they cannot be written in terms of finite, standard mathematical

operations. This is exactly the case with the pandemic equations – with there being no mathematical equation graphing  $S(t)$ ,  $I(t)$  and  $R(t)$ .

This is why the initial part of the code is to create an approximation for these equations – which can be done using a method known as “Euler Integration.” While in the precise formula,  $dt$  refers to an infinitesimally small step in time, we can substitute another small value in for  $dt$ . This gives a new formula, which is defined in terms of a recursive function rather than a differential equation:

$$\begin{aligned} S(t + dt) &= S(t) - (\beta S(t)I(t))dt \\ I(t + dt) &= I(t) + (\beta S(t)I(t) - \gamma I(t))dt \\ R(t + dt) &= R(t) + \gamma I(t)dt \end{aligned}$$

As  $dt$  gets closer and closer to 0, the approximation becomes the true, continuous function. However, for the purpose of this project, a  $dt$  of 1 was chosen. This is a relatively large  $dt$ , making this a limitation of the final product. However, this yields the following equations. ( $I(t)$  is not dependent on  $R(t)$ , this equation can be excluded). Hence, this yields the final recursive equations used to model the pandemic:

$$\begin{aligned} S(t + 1) &= S(t) - \beta S(t)I(t) \\ I(t + 1) &= I(t) + \beta S(t)I(t) - \gamma I(t) \end{aligned}$$

These equations were used to find the spread of the pandemic at integer values of  $t$  (basically whole numbers of days), for 60 days.

It is these equations which can be seen implemented in the following latex form code:

```
S_{0}=1-I_{0}
S_{1}=S_{0}-\left(\beta S_{0}I_{0}\right)
I_{1}=I_{0}+\left(\beta S_{0}I_{0}-\gamma I_{0}\right)
S_{2}=S_{1}-\left(\beta S_{1}I_{1}\right)
I_{2}=I_{1}+\left(\beta S_{1}I_{1}-\gamma I_{1}\right)
S_{3}=S_{2}-\left(\beta S_{2}I_{2}\right)
I_{3}=I_{2}+\left(\beta S_{2}I_{2}-\gamma I_{2}\right)
S_{4}=S_{3}-\left(\beta S_{3}I_{3}\right)
I_{4}=I_{3}+\left(\beta S_{3}I_{3}-\gamma I_{3}\right)
S_{5}=S_{4}-\left(\beta S_{4}I_{4}\right)
I_{5}=I_{4}+\left(\beta S_{4}I_{4}-\gamma I_{4}\right)
S_{6}=S_{5}-\left(\beta S_{5}I_{5}\right)
I_{6}=I_{5}+\left(\beta S_{5}I_{5}-\gamma I_{5}\right)
S_{7}=S_{6}-\left(\beta S_{6}I_{6}\right)
I_{7}=I_{6}+\left(\beta S_{6}I_{6}-\gamma I_{6}\right)
S_{8}=S_{7}-\left(\beta S_{7}I_{7}\right)
I_{8}=I_{7}+\left(\beta S_{7}I_{7}-\gamma I_{7}\right)
S_{9}=S_{8}-\left(\beta S_{8}I_{8}\right)
I_{9}=I_{8}+\left(\beta S_{8}I_{8}-\gamma I_{8}\right)
S_{10}=S_{9}-\left(\beta S_{9}I_{9}\right)
I_{10}=I_{9}+\left(\beta S_{9}I_{9}-\gamma I_{9}\right)
S_{11}=S_{10}-\left(\beta S_{10}I_{10}\right)
I_{11}=I_{10}+\left(\beta S_{10}I_{10}-\gamma I_{10}\right)
S_{12}=S_{11}-\left(\beta S_{11}I_{11}\right)
```

$$L_{12}=I_{11}+\text{left}(\beta S_{11}I_{11})^{-\gamma_{I_{11}}}\text{right}$$

$$S_{13}=S_{12}-\text{left}(\beta S_{12}I_{12})\text{right}$$

$$L_{13}=I_{12}+\text{left}(\beta S_{12}I_{12})^{-\gamma_{I_{12}}}\text{right}$$

$$S_{14}=S_{13}-\text{left}(\beta S_{13}I_{13})\text{right}$$

$$L_{14}=I_{13}+\text{left}(\beta S_{13}I_{13})^{-\gamma_{I_{13}}}\text{right}$$

$$S_{15}=S_{14}-\text{left}(\beta S_{14}I_{14})\text{right}$$

$$L_{15}=I_{14}+\text{left}(\beta S_{14}I_{14})^{-\gamma_{I_{14}}}\text{right}$$

$$S_{16}=S_{15}-\text{left}(\beta S_{15}I_{15})\text{right}$$

$$L_{16}=I_{15}+\text{left}(\beta S_{15}I_{15})^{-\gamma_{I_{15}}}\text{right}$$

$$S_{17}=S_{16}-\text{left}(\beta S_{16}I_{16})\text{right}$$

$$L_{17}=I_{16}+\text{left}(\beta S_{16}I_{16})^{-\gamma_{I_{16}}}\text{right}$$

$$S_{18}=S_{17}-\text{left}(\beta S_{17}I_{17})\text{right}$$

$$L_{18}=I_{17}+\text{left}(\beta S_{17}I_{17})^{-\gamma_{I_{17}}}\text{right}$$

$$S_{19}=S_{18}-\text{left}(\beta S_{18}I_{18})\text{right}$$

$$L_{19}=I_{18}+\text{left}(\beta S_{18}I_{18})^{-\gamma_{I_{18}}}\text{right}$$

$$S_{20}=S_{19}-\text{left}(\beta S_{19}I_{19})\text{right}$$

$$L_{20}=I_{19}+\text{left}(\beta S_{19}I_{19})^{-\gamma_{I_{19}}}\text{right}$$

$$S_{21}=S_{20}-\text{left}(\beta S_{20}I_{20})\text{right}$$

$$L_{21}=I_{20}+\text{left}(\beta S_{20}I_{20})^{-\gamma_{I_{20}}}\text{right}$$

$$S_{22}=S_{21}-\text{left}(\beta S_{21}I_{21})\text{right}$$

$$L_{22}=I_{21}+\text{left}(\beta S_{21}I_{21})^{-\gamma_{I_{21}}}\text{right}$$

$$S_{23}=S_{22}-\text{left}(\beta S_{22}I_{22})\text{right}$$

$$L_{23}=I_{22}+\text{left}(\beta S_{22}I_{22})^{-\gamma_{I_{22}}}\text{right}$$

$$S_{24}=S_{23}-\text{left}(\beta S_{23}I_{23})\text{right}$$

$$L_{24}=I_{23}+\text{left}(\beta S_{23}I_{23})^{-\gamma_{I_{23}}}\text{right}$$

$$S_{25}=S_{24}-\text{left}(\beta S_{24}I_{24})\text{right}$$

$$L_{25}=I_{24}+\text{left}(\beta S_{24}I_{24})^{-\gamma_{I_{24}}}\text{right}$$

$$S_{26}=S_{25}-\text{left}(\beta S_{25}I_{25})\text{right}$$

$$L_{26}=I_{25}+\text{left}(\beta S_{25}I_{25})^{-\gamma_{I_{25}}}\text{right}$$

$$S_{27}=S_{26}-\text{left}(\beta S_{26}I_{26})\text{right}$$

$$L_{27}=I_{26}+\text{left}(\beta S_{26}I_{26})^{-\gamma_{I_{26}}}\text{right}$$

$$S_{28}=S_{27}-\text{left}(\beta S_{27}I_{27})\text{right}$$

$$L_{28}=I_{27}+\text{left}(\beta S_{27}I_{27})^{-\gamma_{I_{27}}}\text{right}$$

$$S_{29}=S_{28}-\text{left}(\beta S_{28}I_{28})\text{right}$$

$$L_{29}=I_{28}+\text{left}(\beta S_{28}I_{28})^{-\gamma_{I_{28}}}\text{right}$$

$$S_{30}=S_{29}-\text{left}(\beta S_{29}I_{29})\text{right}$$

$$L_{30}=I_{29}+\text{left}(\beta S_{29}I_{29})^{-\gamma_{I_{29}}}\text{right}$$

$$S_{31}=S_{30}-\text{left}(\beta S_{30}I_{30})\text{right}$$

$$L_{31}=I_{30}+\text{left}(\beta S_{30}I_{30})^{-\gamma_{I_{30}}}\text{right}$$

$$S_{32}=S_{31}-\text{left}(\beta S_{31}I_{31})\text{right}$$

$$L_{32}=I_{31}+\text{left}(\beta S_{31}I_{31})^{-\gamma_{I_{31}}}\text{right}$$

$$S_{33}=S_{32}-\text{left}(\beta S_{32}I_{32})\text{right}$$

$$L_{\{33\}}=L_{\{32\}}+\text{left}(\text{beta } S_{\{32\}}L_{\{32\}}^{-\text{gamma } I_{\{32\}}}\text{right})$$

$$S_{\{34\}}=S_{\{33\}}-\text{left}(\text{beta } S_{\{33\}}I_{\{33\}}\text{right})$$

$$L_{\{34\}}=L_{\{33\}}+\text{left}(\text{beta } S_{\{33\}}L_{\{33\}}^{-\text{gamma } I_{\{33\}}}\text{right})$$

$$S_{\{35\}}=S_{\{34\}}-\text{left}(\text{beta } S_{\{34\}}I_{\{34\}}\text{right})$$

$$L_{\{35\}}=L_{\{34\}}+\text{left}(\text{beta } S_{\{34\}}L_{\{34\}}^{-\text{gamma } I_{\{34\}}}\text{right})$$

$$S_{\{36\}}=S_{\{35\}}-\text{left}(\text{beta } S_{\{35\}}I_{\{35\}}\text{right})$$

$$L_{\{36\}}=L_{\{35\}}+\text{left}(\text{beta } S_{\{35\}}L_{\{35\}}^{-\text{gamma } I_{\{35\}}}\text{right})$$

$$S_{\{37\}}=S_{\{36\}}-\text{left}(\text{beta } S_{\{36\}}I_{\{36\}}\text{right})$$

$$L_{\{37\}}=L_{\{36\}}+\text{left}(\text{beta } S_{\{36\}}L_{\{36\}}^{-\text{gamma } I_{\{36\}}}\text{right})$$

$$S_{\{38\}}=S_{\{37\}}-\text{left}(\text{beta } S_{\{37\}}I_{\{37\}}\text{right})$$

$$L_{\{38\}}=L_{\{37\}}+\text{left}(\text{beta } S_{\{37\}}L_{\{37\}}^{-\text{gamma } I_{\{37\}}}\text{right})$$

$$S_{\{39\}}=S_{\{38\}}-\text{left}(\text{beta } S_{\{38\}}I_{\{38\}}\text{right})$$

$$L_{\{39\}}=L_{\{38\}}+\text{left}(\text{beta } S_{\{38\}}L_{\{38\}}^{-\text{gamma } I_{\{38\}}}\text{right})$$

$$S_{\{40\}}=S_{\{39\}}-\text{left}(\text{beta } S_{\{39\}}I_{\{39\}}\text{right})$$

$$L_{\{40\}}=L_{\{39\}}+\text{left}(\text{beta } S_{\{39\}}L_{\{39\}}^{-\text{gamma } I_{\{39\}}}\text{right})$$

$$S_{\{41\}}=S_{\{40\}}-\text{left}(\text{beta } S_{\{40\}}I_{\{40\}}\text{right})$$

$$L_{\{41\}}=L_{\{40\}}+\text{left}(\text{beta } S_{\{40\}}L_{\{40\}}^{-\text{gamma } I_{\{40\}}}\text{right})$$

$$S_{\{42\}}=S_{\{41\}}-\text{left}(\text{beta } S_{\{41\}}I_{\{41\}}\text{right})$$

$$L_{\{42\}}=L_{\{41\}}+\text{left}(\text{beta } S_{\{41\}}L_{\{41\}}^{-\text{gamma } I_{\{41\}}}\text{right})$$

$$S_{\{43\}}=S_{\{42\}}-\text{left}(\text{beta } S_{\{42\}}I_{\{42\}}\text{right})$$

$$L_{\{43\}}=L_{\{42\}}+\text{left}(\text{beta } S_{\{42\}}L_{\{42\}}^{-\text{gamma } I_{\{42\}}}\text{right})$$

$$S_{\{44\}}=S_{\{43\}}-\text{left}(\text{beta } S_{\{43\}}I_{\{43\}}\text{right})$$

$$L_{\{44\}}=L_{\{43\}}+\text{left}(\text{beta } S_{\{43\}}L_{\{43\}}^{-\text{gamma } I_{\{43\}}}\text{right})$$

$$S_{\{45\}}=S_{\{44\}}-\text{left}(\text{beta } S_{\{44\}}I_{\{44\}}\text{right})$$

$$L_{\{45\}}=L_{\{44\}}+\text{left}(\text{beta } S_{\{44\}}L_{\{44\}}^{-\text{gamma } I_{\{44\}}}\text{right})$$

$$S_{\{46\}}=S_{\{45\}}-\text{left}(\text{beta } S_{\{45\}}I_{\{45\}}\text{right})$$

$$L_{\{46\}}=L_{\{45\}}+\text{left}(\text{beta } S_{\{45\}}L_{\{45\}}^{-\text{gamma } I_{\{45\}}}\text{right})$$

$$S_{\{47\}}=S_{\{46\}}-\text{left}(\text{beta } S_{\{46\}}I_{\{46\}}\text{right})$$

$$L_{\{47\}}=L_{\{46\}}+\text{left}(\text{beta } S_{\{46\}}L_{\{46\}}^{-\text{gamma } I_{\{46\}}}\text{right})$$

$$S_{\{48\}}=S_{\{47\}}-\text{left}(\text{beta } S_{\{47\}}I_{\{47\}}\text{right})$$

$$L_{\{48\}}=L_{\{47\}}+\text{left}(\text{beta } S_{\{47\}}L_{\{47\}}^{-\text{gamma } I_{\{47\}}}\text{right})$$

$$S_{\{49\}}=S_{\{48\}}-\text{left}(\text{beta } S_{\{48\}}I_{\{48\}}\text{right})$$

$$L_{\{49\}}=L_{\{48\}}+\text{left}(\text{beta } S_{\{48\}}L_{\{48\}}^{-\text{gamma } I_{\{48\}}}\text{right})$$

$$S_{\{50\}}=S_{\{49\}}-\text{left}(\text{beta } S_{\{49\}}I_{\{49\}}\text{right})$$

$$L_{\{50\}}=L_{\{49\}}+\text{left}(\text{beta } S_{\{49\}}L_{\{49\}}^{-\text{gamma } I_{\{49\}}}\text{right})$$

$$S_{\{51\}}=S_{\{50\}}-\text{left}(\text{beta } S_{\{50\}}I_{\{50\}}\text{right})$$

$$L_{\{51\}}=L_{\{50\}}+\text{left}(\text{beta } S_{\{50\}}L_{\{50\}}^{-\text{gamma } I_{\{50\}}}\text{right})$$

$$S_{\{52\}}=S_{\{51\}}-\text{left}(\text{beta } S_{\{51\}}I_{\{51\}}\text{right})$$

$$L_{\{52\}}=L_{\{51\}}+\text{left}(\text{beta } S_{\{51\}}L_{\{51\}}^{-\text{gamma } I_{\{51\}}}\text{right})$$

$$S_{\{53\}}=S_{\{52\}}-\text{left}(\text{beta } S_{\{52\}}I_{\{52\}}\text{right})$$

$$L_{\{53\}}=L_{\{52\}}+\text{left}(\text{beta } S_{\{52\}}L_{\{52\}}^{-\text{gamma } I_{\{52\}}}\text{right})$$

$$S_{\{54\}}=S_{\{53\}}-\text{left}(\text{beta } S_{\{53\}}I_{\{53\}}\text{right})$$

$$I_{\{54\}}=I_{\{53\}}+\text{left}(\text{beta } S_{\{53\}}I_{\{53\}}-\text{gamma } I_{\{53\}}\text{right})$$

$$S_{\{55\}}=S_{\{54\}}-\text{left}(\text{beta } S_{\{54\}}I_{\{54\}}\text{right})$$

$$I_{\{55\}}=I_{\{54\}}+\text{left}(\text{beta } S_{\{54\}}I_{\{54\}}-\text{gamma } I_{\{54\}}\text{right})$$

$$S_{\{56\}}=S_{\{55\}}-\text{left}(\text{beta } S_{\{55\}}I_{\{55\}}\text{right})$$

$$I_{\{56\}}=I_{\{55\}}+\text{left}(\text{beta } S_{\{55\}}I_{\{55\}}-\text{gamma } I_{\{55\}}\text{right})$$

$$S_{\{57\}}=S_{\{56\}}-\text{left}(\text{beta } S_{\{56\}}I_{\{56\}}\text{right})$$

$$I_{\{57\}}=I_{\{56\}}+\text{left}(\text{beta } S_{\{56\}}I_{\{56\}}-\text{gamma } I_{\{56\}}\text{right})$$

$$S_{\{58\}}=S_{\{57\}}-\text{left}(\text{beta } S_{\{57\}}I_{\{57\}}\text{right})$$

$$I_{\{58\}}=I_{\{57\}}+\text{left}(\text{beta } S_{\{57\}}I_{\{57\}}-\text{gamma } I_{\{57\}}\text{right})$$

$$S_{\{59\}}=S_{\{58\}}-\text{left}(\text{beta } S_{\{58\}}I_{\{58\}}\text{right})$$

$$I_{\{59\}}=I_{\{58\}}+\text{left}(\text{beta } S_{\{58\}}I_{\{58\}}-\text{gamma } I_{\{58\}}\text{right})$$

$$S_{\{60\}}=S_{\{59\}}-\text{left}(\text{beta } S_{\{59\}}I_{\{59\}}\text{right})$$

$$I_{\{60\}}=I_{\{59\}}+\text{left}(\text{beta } S_{\{59\}}I_{\{59\}}-\text{gamma } I_{\{59\}}\text{right})$$

These points, for  $I(t)$  were then graphed, in the following table, which can also be seen in the final Desmos link:

T	I
0	$I_{\{0\}}$
1	$I_{\{1\}}$
2	$I_{\{2\}}$
3	$I_{\{3\}}$
4	$I_{\{4\}}$
5	$I_{\{5\}}$
6	$I_{\{6\}}$
7	$I_{\{7\}}$
8	$I_{\{8\}}$
9	$I_{\{9\}}$
10	$I_{\{10\}}$
11	$I_{\{11\}}$
12	$I_{\{12\}}$
13	$I_{\{13\}}$
14	$I_{\{14\}}$
15	$I_{\{15\}}$
16	$I_{\{16\}}$
17	$I_{\{17\}}$
18	$I_{\{18\}}$
19	$I_{\{19\}}$
20	$I_{\{20\}}$
21	$I_{\{21\}}$
22	$I_{\{22\}}$
23	$I_{\{23\}}$
24	$I_{\{24\}}$
25	$I_{\{25\}}$
26	$I_{\{26\}}$
27	$I_{\{27\}}$
28	$I_{\{28\}}$
29	$I_{\{29\}}$
30	$I_{\{30\}}$
31	$I_{\{31\}}$
32	$I_{\{32\}}$
33	$I_{\{33\}}$
34	$I_{\{34\}}$
35	$I_{\{35\}}$
36	$I_{\{36\}}$
37	$I_{\{37\}}$
38	$I_{\{38\}}$
39	$I_{\{39\}}$
40	$I_{\{40\}}$
41	$I_{\{41\}}$
42	$I_{\{42\}}$
43	$I_{\{43\}}$
44	$I_{\{44\}}$
45	$I_{\{45\}}$
46	$I_{\{46\}}$
47	$I_{\{47\}}$
48	$I_{\{48\}}$
49	$I_{\{49\}}$
50	$I_{\{50\}}$
51	$I_{\{51\}}$



52	I_{52}
53	I_{53}
54	I_{54}
55	I_{55}
56	I_{56}
57	I_{57}
58	I_{58}
59	I_{59}
60	I_{60}

The next step in the approximation, was to “fill in” the spaces between the points – and to approximate the spread of the pandemic at non-integer values of  $t$ . To do this, a process known as a “Fourier Series” can be used. This would approximate the function as a sum of sine waves, giving a purely sinusoidal approximation for  $I(t)$ .

The formula for a Fourier series for  $I(t)$  on the interval 0 to  $2l$  is:

$$I(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left( a_n \cos\left(\frac{n\pi t}{l}\right) + b_n \sin\left(\frac{n\pi t}{l}\right) \right)$$

Where:

$$a_0 = \frac{1}{l} \int_0^{2l} I(t) dt$$

$$a_n = \frac{1}{l} \int_0^{2l} I(t) \cos\left(\frac{n\pi t}{l}\right) dt$$

$$b_n = \frac{1}{l} \int_0^{2l} I(t) \sin\left(\frac{n\pi t}{l}\right) dt$$

Note that this formula includes taking the integral of the function, which cannot be done, as  $I(t)$  is not continuous! It is in this case, where the function’s integral can be approximated with a series of rectangles, which the viewer can view as shown in Figure 03. The code for the visual representation of these rectangles, in latex form, are:

```

\operatorname{polygon}\left(\left(0,0\right),\left(0,I_{0}\right),\left(1,I_{0}\right),\left(1,0\right)\right)
\operatorname{polygon}\left(\left(1,0\right),\left(1,I_{1}\right),\left(2,I_{1}\right),\left(2,0\right)\right)
\operatorname{polygon}\left(\left(2,0\right),\left(2,I_{2}\right),\left(3,I_{2}\right),\left(3,0\right)\right)
\operatorname{polygon}\left(\left(3,0\right),\left(3,I_{3}\right),\left(4,I_{3}\right),\left(4,0\right)\right)
\operatorname{polygon}\left(\left(4,0\right),\left(4,I_{4}\right),\left(5,I_{4}\right),\left(5,0\right)\right)
\operatorname{polygon}\left(\left(5,0\right),\left(5,I_{5}\right),\left(6,I_{5}\right),\left(6,0\right)\right)
\operatorname{polygon}\left(\left(6,0\right),\left(6,I_{6}\right),\left(7,I_{6}\right),\left(7,0\right)\right)
\operatorname{polygon}\left(\left(7,0\right),\left(7,I_{7}\right),\left(8,I_{7}\right),\left(8,0\right)\right)
\operatorname{polygon}\left(\left(8,0\right),\left(8,I_{8}\right),\left(9,I_{8}\right),\left(9,0\right)\right)
\operatorname{polygon}\left(\left(9,0\right),\left(9,I_{9}\right),\left(10,I_{9}\right),\left(10,0\right)\right)
\operatorname{polygon}\left(\left(10,0\right),\left(10,I_{10}\right),\left(11,I_{10}\right),\left(11,0\right)\right)
\operatorname{polygon}\left(\left(11,0\right),\left(11,I_{11}\right),\left(12,I_{11}\right),\left(12,0\right)\right)
\operatorname{polygon}\left(\left(12,0\right),\left(12,I_{12}\right),\left(13,I_{12}\right),\left(13,0\right)\right)
\operatorname{polygon}\left(\left(13,0\right),\left(13,I_{13}\right),\left(14,I_{13}\right),\left(14,0\right)\right)
\operatorname{polygon}\left(\left(14,0\right),\left(14,I_{14}\right),\left(15,I_{14}\right),\left(15,0\right)\right)

```



$\operatorname{polygon}\left(\left(59,0\right),\left(59,1_{59}\right),\left(60,1_{59}\right),\left(60,0\right)\right)$

As more and more sinusoidal terms are added in the series, this approaches a perfect interpolation of the function. However, for the purpose of the program, only 20 different sine and cosine functions of different frequencies were added. This gives quite a good approximation of  $I(t)$  for most initial conditions, however sometimes the sinusoidal nature of the approximation can be seen. This is further shown on the final product. The code in latex, for the final Fourier Series was is shown below. Note that this is written as a function of  $x$ , even though the variable initially assigned was  $t$ . This is because Desmos only supports functions of  $x$ :

$$f(x)=\frac{a_0}{2}+a_1\cos\left(\frac{\pi x}{30}\right)+b_1\sin\left(\frac{\pi x}{30}\right)+a_2\cos\left(\frac{2\pi x}{30}\right)+b_2\sin\left(\frac{2\pi x}{30}\right)+a_3\cos\left(\frac{3\pi x}{30}\right)+b_3\sin\left(\frac{3\pi x}{30}\right)+a_4\cos\left(\frac{4\pi x}{30}\right)+b_4\sin\left(\frac{4\pi x}{30}\right)+a_5\cos\left(\frac{5\pi x}{30}\right)+b_5\sin\left(\frac{5\pi x}{30}\right)+a_6\cos\left(\frac{6\pi x}{30}\right)+b_6\sin\left(\frac{6\pi x}{30}\right)+a_7\cos\left(\frac{7\pi x}{30}\right)+b_7\sin\left(\frac{7\pi x}{30}\right)+a_8\cos\left(\frac{8\pi x}{30}\right)+b_8\sin\left(\frac{8\pi x}{30}\right)+a_9\cos\left(\frac{9\pi x}{30}\right)+b_9\sin\left(\frac{9\pi x}{30}\right)+a_{10}\cos\left(\frac{10\pi x}{30}\right)+b_{10}\sin\left(\frac{10\pi x}{30}\right)$$

$a_0=\frac{1}{30}$

$a_1=\frac{1}{30}\cos\left(\frac{\pi T}{30}\right)$

$b_1=\frac{1}{30}\sin\left(\frac{\pi T}{30}\right)$

$a_2=\frac{1}{30}\cos\left(\frac{2\pi T}{30}\right)$

$b_2=\frac{1}{30}\sin\left(\frac{2\pi T}{30}\right)$

$a_3=\frac{1}{30}\cos\left(\frac{3\pi T}{30}\right)$

$b_3=\frac{1}{30}\sin\left(\frac{3\pi T}{30}\right)$

$a_4=\frac{1}{30}\cos\left(\frac{4\pi T}{30}\right)$

$b_4=\frac{1}{30}\sin\left(\frac{4\pi T}{30}\right)$

$a_5=\frac{1}{30}\cos\left(\frac{5\pi T}{30}\right)$

$b_5=\frac{1}{30}\sin\left(\frac{5\pi T}{30}\right)$

$a_6=\frac{1}{30}\cos\left(\frac{6\pi T}{30}\right)$

$b_6=\frac{1}{30}\sin\left(\frac{6\pi T}{30}\right)$

$a_7=\frac{1}{30}\cos\left(\frac{7\pi T}{30}\right)$

$b_7=\frac{1}{30}\sin\left(\frac{7\pi T}{30}\right)$

$a_8=\frac{1}{30}\cos\left(\frac{8\pi T}{30}\right)$

$b_8=\frac{1}{30}\sin\left(\frac{8\pi T}{30}\right)$

$a_9=\frac{1}{30}\cos\left(\frac{9\pi T}{30}\right)$

$b_9=\frac{1}{30}\sin\left(\frac{9\pi T}{30}\right)$

$a_{10}=\frac{1}{30}\cos\left(\frac{10\pi T}{30}\right)$

$b_{10}=\frac{1}{30}\sin\left(\frac{10\pi T}{30}\right)$

Similarly, it is important to see the rate of change of the pandemic at any given moment. For this, the derivative of the Fourier series can be found, which approximates the derivative of  $I(t)$ . To implement this as can be seen in Figure 04, the following latex code can be used:

$k=$

$$g(x)=\frac{d}{dx}f(x)$$

$$g(k)=f(x-k)+f(k)$$

$$x=k\left(\frac{g(k)}{f(k)}\right)$$

In these equations,  $k$  is some value of  $t$ , which the user can use the sliders to change – visualising the derivative of  $I(t)$  at that point.

Finally, the highest point of the original points can be written as:

$\max\left(I\right)$

And the places where the derivative of the Fourier series is equal to 0 is represented in the following code, which show the local maxima and minima:

$\frac{d}{dx}f\left(x\right)=0$

## Acknowledgements

I would like to acknowledge a few very good sources and YouTube videos, which greatly assisted in helping me devise the formulae I ended up using. I would also like to acknowledge a Desmos online graphing calculator, for making this project possible.

## Bibliography

- [www.youtube.com](https://www.youtube.com/watch?v=Qrp40ck3WpI&t=613s). (n.d.). *The MATH of Pandemics / Intro to the SIR Model*. [online] Available at: <https://www.youtube.com/watch?v=Qrp40ck3WpI&t=613s>
- [www.youtube.com](https://www.youtube.com/watch?v=2Hsl1C1ov5c&t=365s). (n.d.). *SIR Model For Disease Spread- 3. Euler Integration in Spreadsheets*. [online] Available at: <https://www.youtube.com/watch?v=2Hsl1C1ov5c&t=365s>
- [www.youtube.com](https://www.youtube.com/watch?v=Dnf8vahAzDI&t=130s). (n.d.). *Fourier Series from 0 to 2L*. [online] Available at: <https://www.youtube.com/watch?v=Dnf8vahAzDI&t=130s>
- Mathematics LibreTexts. (2022). *4.3: The SIR Epidemic Disease Model*. [online] Available at: [https://math.libretexts.org/Bookshelves/Applied\\_Mathematics/Mathematical\\_Biology\\_\(Chasnov\)/04%3A\\_Infectious\\_Disease\\_Modeling/4.03%3A\\_The\\_SIR\\_Epidemic\\_Disease\\_Model](https://math.libretexts.org/Bookshelves/Applied_Mathematics/Mathematical_Biology_(Chasnov)/04%3A_Infectious_Disease_Modeling/4.03%3A_The_SIR_Epidemic_Disease_Model)
- Smith, D. and Moore, L. (2015). *The SIR Model for Spread of Disease - The Differential Equation Model* | Mathematical Association of America. [online] Maa.org. Available at: <https://www.maa.org/press/periodicals/loci/joma/the-sir-model-for-spread-of-disease-the-differential-equation-model>.